Biology By Numbers

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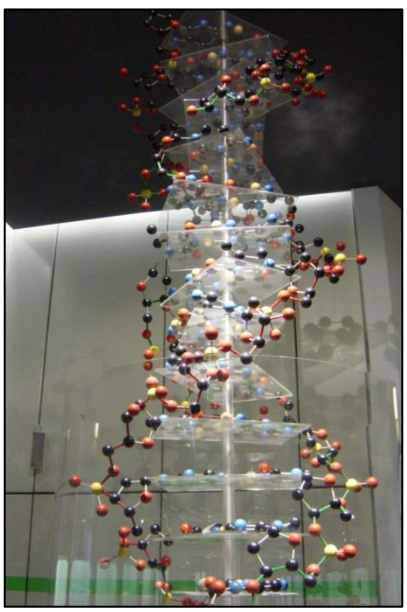
It’s in Your Genes: Tree Diagrams, Probability & Inheritance

BY NIMBIOS January 31, 2012 NO COMMENTS



Once upon a time Blaise Pascal, a French mathematician, was asked by a nobleman for some help with understanding a favorite dice game of chance. This conversation carried into some correspondence with another mathematician friend named de Méré, and sparked the beginning of probability theory as we know it today. Little did Pascal and de Méré know that the same rules of probability that govern throwing a die would also apply to the traits of offspring when two parents get together. It wasn’t until about 200 years later that Gregor Mendel would start his work with pea plants and became founder of the field of genetics. Genetics represents a beautiful illustration of the unity of mathematics and biology. In this exercise, poker chips are used to represent genes, and drawing tree diagrams help to illustrate the chance for inheritance of multiple traits. This activity is borrowed with permission from Biology in a Box (link=http://eeb.bio.utk.edu/biologyinbox/)

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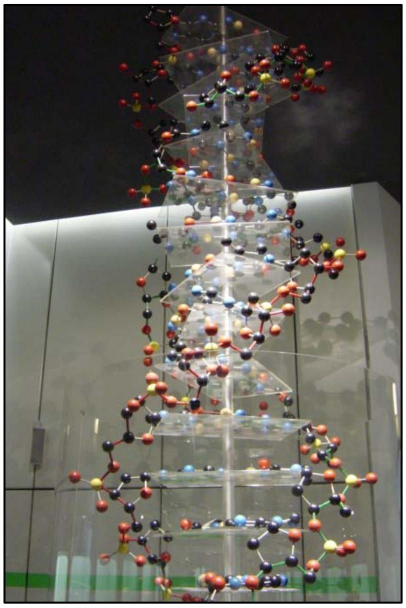
It's in Your Genes: Tree Diagrams, Probability & Inheritance



Grade Level: 9th-12th grade
Subject Matter: Genetics, Probability, Mathematics

Vocabulary
Trait: a genetically determined characteristic
Chromosome: a single piece of DNA containing information for many genes as well as proteins, an organized structure located in the cell nucleus
Allele: one of two or more alternate forms of a gene, located on the same place on the chromosome
Inheritance: the passing on of gene alleles from parents to offspring
Genotype: the specific allele make-up of an organism or cell
Probability: the likelihood an event will occur
Independent Events: two events (occurances) that may happen that do not depend on the other to occur
Random: equal chance that any possible event may occur

One can use probability to analyze the way an offspring might inherit a *single* trait, where there are only two steps or *tasks*. The first task is to randomly pick the allele that came from the female parent, and the second task was to randomly pick the allele that came from the male parent. It is easy to list and count all of the possible outcomes. In considering how an offspring might inherit *multiple* traits, in which many more tasks (with many more outcomes) are involved, using tree diagrams and the counting principle is another helpful way to determine the probability of an event, and an alternative to drawing a Punnett square.

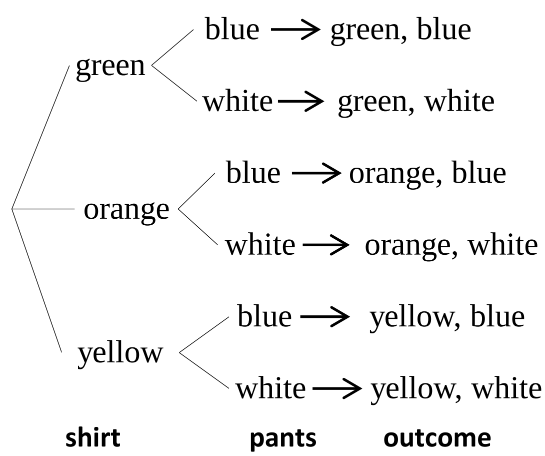
The counting principle: If there are x ways to perform one task, and y ways to perform a second task, then there are xy ways to perform both tasks.

Suppose for example that we flip two coins. Then there are 2 possible results from the flip of the first coin (heads or tails), and 2 possible results from the flip of the second coin, so there are $2 \times 2 = 4$ possible results when flipping both coins.

1. Suppose that you have three shirts (green, orange, and yellow) and two pairs of pants (blue and white). How many different outfits can you assemble from these clothing items?

Tree diagrams

- A tree diagram is a visual aid that helps us find and count all possible outcomes of an experiment. For example, we can construct a tree diagram for the experiment from question 1.



- A tree diagram can also help us to find the probability that an event occurs. For example, suppose that when you get up in the morning, you grab a shirt from your closet without turning on the light. **What is the probability that your shirt is yellow?**

- **Answer:** Since you don't turn on the light, you are equally likely to grab any of the shirts in your closet. Therefore, the probability that your shirt is yellow is the quotient of the number of outcomes with a yellow shirt divided by the total number of outcomes. Looking at the tree diagram we see that there are two outcomes with a yellow shirt and six outcomes total, so the probability that your shirt is yellow is 2/6 = 1/3.

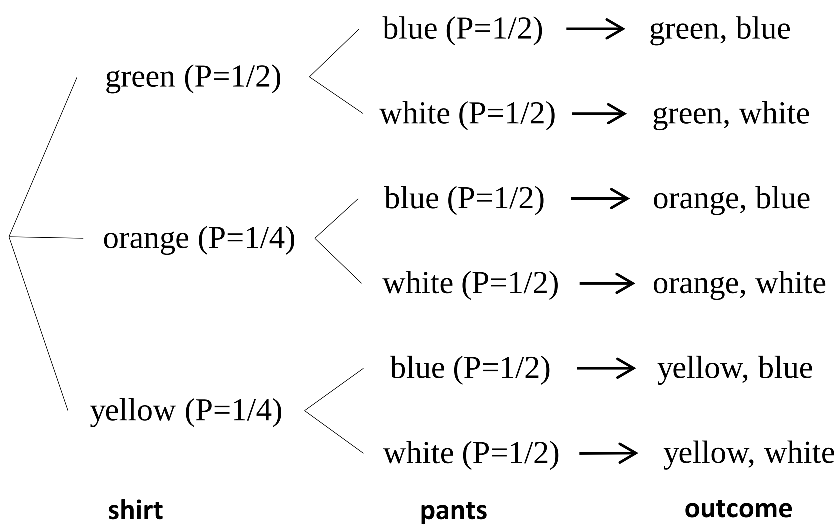
2. What is the probability that your shirt is yellow *or* orange?

What if you got another green shirt as a birthday gift? This would change the probabilities of each outcome, but we can still use a tree diagram to help us do so, using the rules of probability. Now you would have four shirts instead of three, so the probabilities of getting each possible outfit would change, since the probabilities of grabbing a shirt of a particular color are no longer all equal.

- **What are the probabilities of getting each color shirt now that you have received this gift?**

- **Answer:** The probability that you would draw a green shirt at random now would be equal to 2/4 = 1/2, since you now have four shirts, and two of them are green. The probability of drawing an orange shirt would now be equal to 1/4, and the probability of drawing a yellow shirt would also be 1/4.

Though you could have reflected this by drawing another tree diagram, and adding an additional branch to represent the new green shirt, another way of calculating the probabilities of the overall outcome (the final outfit) without having to draw extra branches would be to write the probabilities of the result of each task on our tree diagram. We can also then use these to calculate the probabilities of each of the outcomes. Look at the new tree diagram on the following page for an example.



Recall the counting principle: that if events A and B are independent, $P(A \text{ } and \text{ } B) = P(A)P(B)$. Since the shirt that you grab from your closet at random has no effect on the pants that you grab at random (these events are independent!), you can calculate the probabilities of each outcome on your tree diagram by simply multiplying the probabilities of the branches along the branches leading to that particular outcome, as shown below:

shirt	pants	outcome
green (P=1/2)	blue (P=1/2) →	green, blue (P=1/2 × 1/2 = 1/4)
	white (P=1/2) →	green, white (P=1/2 × 1/2 = 1/4)
orange (P=1/4)	blue (P=1/2) →	orange, blue (P=1/4 × 1/2 = 1/8)
	white (P=1/2) →	orange, white (P=1/4 × 1/2= 1/8)
yellow (P=1/4)	blue (P=1/2) →	yellow, blue (P= 1/4 × 1/2 = 1/8)
	white (P=1/2) →	yellow, white (P=1/4 × 1/2 = 1/8)

Now, let's apply this information to the topic at hand, the inheritance of multiple traits. Think of a cross between two parents, both with the genotype ***CC'TT'***. ***C and C'*** are two different alleles for color, and ***T and T'*** are different alleles for size.

3. How many different tasks are involved in determining the genotype of an offspring in this cross? List each of these tasks.

4. In how many ways can each of the tasks from the previous question be performed?

5. Construct a tree diagram to show all of the possible outcomes of this cross.

6. How many outcomes are possible for this experiment?

7. How many possible genotypes could the offspring produced by this cross have? (Remember that the genotype is the set of alleles that an individual has with no regard to which parent donated which allele.)

Activity Materials

- One "Female Parent" Box and One "Male Parent" Box per group
- Poker chips – 2 colors, 2 thicknesses, 3 of each (total of 12) per box
- Blindfold

What to do

- Now we will simulate the inheritance of multiple traits by an offspring produced by a cross between parents of randomly determined genotypes.

- Find the boxes labeled **Female** (representing the female parent's gene pool) and **Male** (representing the male parent's gene pool).
- Make sure each box has equal numbers of thick red, thin red, thick white, and thin white chips (3 of each)..
- Put on your blindfold and find your parent's genotypes by picking two chips each from the respective **Female** and **Male** boxes. Record the genotypes of the parents on a sheet of paper.
- Repeat this experiment two more times, simulating new parent genotypes for each cross as described above.
- Answer **3-7** above for each of your crosses.

8. Let A be the event that the offspring has at least one $\boldsymbol{C}$ allele, and B be the event that the offspring has at least one $\boldsymbol{T}$ allele. Use each of your tree diagrams to find $P(A)$, $P(B)$, $P(A \text{ and } B)$, and $P(A)P(B)$ from each of your crosses. Do your answers support Mendel's *Law of Independent Assortment*? That is, do they support the hypothesis that alleles for the color and thickness genes are inherited independently? Explain why or why not.

9. (Critical thinking!) Why was it important that you made sure that both the F and M boxes contained equal numbers of thick red, thin red, thick white, and thin white chips?

Answers

1. $3 \times 2 = 6$

2. $\frac{4}{6} = \frac{2}{3}$

3. There are four tasks involved in the determination of an offspring's genotype:
   1. The female parent must donate an allele for color.
   2. The female parent must donate an allele for size.
   3. The male parent must donate an allele for color.
   4. The male parent must donate an allele for size.

You could have also said that there were two tasks involved: the production of a gamete by the female parent, and the production of a gamete by the male parent. However, just keep in mind that production of each gamete includes the donation of an allele for both color and size, which would mean that each of the "gamete production" tasks could have more possible ways that they could be performed, each involving a combination of size and color alleles!

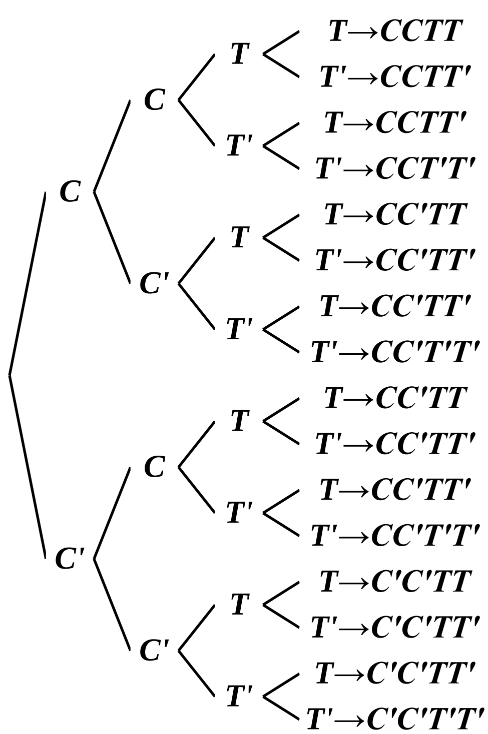
4. This question could be answered this way …

    1. Female parent's donation of color allele: 2 ways (*C* or *C'*)
    2. Male parent's donation of color allele: 2 ways (*C* or *C'*)
    3. Female parent's donation of size allele: 2 ways (*T* or *T'*)
    4. Male parent's donation of size allele: 2 ways (*T* or *T'*)

    Or, if you considered this to be two tasks, the production of a gamete by each parent, the total number of ways each task can be performed are as follows:

    1. Female parent's gamete produced: 4 ways (*CT*, *CT'*, *C'T*, *C'T'*)
    2. Male parent's gamete produced: 4 ways (*CT*, *CT'*, *C'T*, *C'T'*)

5. In the tree diagram on the next page, the first branch point represents the possible alleles for color donated by one parent. The next set of branches represents the possible alleles for color donated by the other parent, given the allele for color donated by the first parent. The next set of branches represents possible alleles for thickness donated by one parent, given the alleles donated by both parents for color, and the final set of branches represents the possible alleles for thickness donated by the other parent, given the alleles donated by both parents for color, and the allele for thickness donated by the first parent. Finally, the outcomes after the arrows represent the resulting genotypes of the offspring in each scenario.



6. There are a total of 16 possible outcomes for this experiment.

7. As you can see in the tree diagram above, even though there are sixteen possible outcomes, some of those outcomes are the same, but just represent different ways that those outcomes could

have occurred. There are only nine possible offspring genotypes (*CCTT*, *CCTT'*, *CCT'T'*, *CC'TT*, *CC'TT'*, *CC'T'T'*, *C'C'TT*, *C'C'TT'*, *C'C'T'T'*) from the cross of two parents both with the genotype *CC'TT'*, but some of these outcomes are more likely than others!

8. You should have found in all cases that $P(A)P(B) = P(A \text{ and } B)$. Recall that this is equivalent to the statement that the events A and B are independent.

9. The reason that it was important for there to be equal numbers of each of these types of chips is because if there had been, for example, one less thick white chip, this would have represented a situation where the two traits (color and size) did NOT assort independently, which was not the point of this exercise! In other words, if you were one thick white chip short, this would have represented that the red (C) and thick (T) alleles are more likely to go together during gamete production (which does happen sometimes with certain genes!).

Extended Links:
Khan Academy's Intro to Heredity

http://www.youtube.com/watch?v=eEUvRrhmcxM&feature=relmfu

The Tech Museum's Understanding Genetics Page: includes a calculator for determining the probability of your offspring's eye color

http://www.thetech.org/genetics/